MAS224, Actuarial Mathematics: Worked examples on annuities (Lecture 9)

1. A loan of £300000 is to be repaid by 10 equal annual installments, the first is due now. Find the annual payment if the interest on the loan is charged at the rate 12.99% per annum.

Solution

Let £C be the annual payment. Have an annuity-due payable annually at rate £C per annum with the present value $C\ddot{a}_{\overline{10|}}$

Equating the P.V.s we we obtain desired equation for C:

$$300000 = C\ddot{a}_{\overline{10|}}, \qquad \ddot{a}_{\overline{n|}} = \frac{1-v^n}{1-v}.$$

Therefore

$$C = \frac{300000}{\ddot{a}_{\overline{10|}}} = \frac{300000(1-v)}{1-v^{10}}\Big|_{@i=0.1299} = £48911.2067 \quad \text{(to 4 d.p.)}$$

The annual repayment is $£48911.21$.

2. Mr. Smith is buying a plasma TV with a cash price of £1499 and is being offered a credit agreement. The terms of the agreement are that £299 is to be paid down on the day of purchase and the remaining amount is to be repaid by 24 equal monthly repayments, the first installment being due one month from the day of purchase. If the APR charged on this credit agreement is 22.95%, what is the monthly repayment?

Solution:

The amount outstanding is £(1499-299)=£1200.

Let £C be the monthly repayment. Have an immediate annuity payable monthly at the rate of £$12C$ per annum.

Equating the P.V.s we obtain the desired equation for C:

$$1200 = 12Ca^{(12)}_{\overline{2|}}, \qquad a^{(p)}_{\overline{n|}} = \frac{1}{p}\frac{v^{1/p}(1-v^n)}{1-v^{1/p}}.$$

From this, $C = \frac{1200}{12a^{(12)}_{\overline{2|}}}$. Evaluating this at $v = \frac{1}{1+i} = \frac{1}{1.2295}$, obtain $C = 61.5683$ (to 4 d.p.).

Hence the monthly repayment is £61.57 per month.

3. A borrower is under an obligation to repay a bank £4300 in two years' time and £1200 in three years' time from now. As part of a review of his future commitments the borrower now offers to discharge his liability for these two debts

(Option 1) either by making an appropriate single payment two years from now,

(Option 2) or by 24 appropriate equal payments made at one month' intervals, the first payment being due now.

Assuming the annual effective rate of 12%, (a) find the appropriate single payment for Option 1 and (b) the appropriate monthly repayment for Option 2.

Solution

(a) Option 1 - single payment

Suppose that the liability is being discharged by a single payment of £C in two years' time from now. Then its P.V. is $\pounds Cvv^2$.

The total P.V. of loans is $\pounds 4300v^2 + \pounds 1200v^3$.

Equating the present values (at time $t = 0$) obtain the desired equation for C:

$$Cv^2 = 4300v^2 + 1200v^3.$$

(Can also equate the P.V.s at time $t = 2$, $C = 4300 + 1200v$.)

As $v = \frac{1}{1+i}|_{@i=0.12}$, we obtain that $C = 4300 + 1200\frac{1}{1.12} = 5371.4286$ (to 4 d.p.).

Therefore, for bank to accept Option 1, the single payment must be must be £5371.43.

(b) Option 2 - annuity-due

Let £C be the amount to be paid monthly. Then the liability is being discharged by an annuity-due payable monthly for two years at rate £$12C$ per annum. Its present value is $12C\ddot{a}^{(12)}_{\overline{24}|}$.

The total P.V. of loans is $\pounds 4300v^2 + \pounds 1200v^3$ (as before). Equating the present values (at time $t = 0$):

$$12C\ddot{a}^{(12)}_{\overline{2}|} = 4300v^2 + 1200v^3, \qquad \ddot{a}^{(12)}_{\overline{2}|} = \frac{1}{p}\frac{(1 - v^n)}{(1 - v^{1/p})}, \qquad v = \frac{1}{1+i}\bigg|_{@i=0.12} = \frac{1}{1.12}$$

and, therefore,

$$C = \frac{4300v^2 + 1200v^3}{12\,\ddot{a}^{(12)}_{\overline{2}|}} = 198.4642 \quad \text{(to 4 d.p.)}$$

£198.46 to be paid monthly